**Problem Sheet 1**

**Question 1**

a)

i)

Let A = Alice goes to the cinema

Let B = Alice goes to the Theatre

ii)

Let A = Two is a prime number

Let B = Two is an odd number

iii)

Let A = Speed limit is 30mph

Let B = I am driving at 25mph

Let C = I am not breaking the law

iv)

Let A = Bob is sleeping

Let B = Bob is working

Let C = Bob is eating

Let D = Bob is relaxing

v)

Let A = Carlos will go to the park

Let B = It is raining

vi)

Ax = x is mortal

vii)

Ax = x is unicorn

viii)

Ax = x is a professional tennis player

By = x is an amateur player

Cxy = x can beat y

ix)

Axy = y is the father of x

Bxy = y is the mother of x

b)

i)

A = I will go to the shops

B = I will go out for lunch

C = My partner will be unhappy

If I either go to the shops and go out for lunch or if I don’t do either, then my partner will be unhappy.

ii)

A = True

B = False

C = True

When A is true, and B is false then the below expression outputs false.

Since the output of the above expression is false then C must be true, leading to the entire expression to be true, hence P is true.

iii)

For P to be false, the following expression must be true.

Hence, C must be false. For this to be the case then either A and B must be true, and C must be False. Alternatively, both A and B can be false, leading to C to be false.

c)

i)

Q is true. Only one animal needs to be sleeping for the expression to be satisfied, which it is with. Since is true, then is true. being true confirms that all animals in the zoo must be sleeping.

ii)

Q is still true as the condition only requires one animal to be sleeping to conclude that all animals are sleeping. Although this is contradictory. However, is true and the contradiction is somewhat overridden by being true, leading to Q to be true.

iii)

There is no confirmation that an animal is sleeping as all could be awake. Since we do not have the information that an animal is sleeping, we cannot know the value of . Without knowing the value of , then we cannot know the value of Q.

iv)

Again, like the previous answer, we do not have the information of whether an animal is sleeping or not, hence, we cannot determine the value of and therefore we cannot determine the value of Q.

**Question 2**

a)

i)

*Reflexive* – Yes as every animal is the same species as itself.

*Symmetric* – Yes as if animal a is the same species as animal b, therefore animal b must be the same species as animal a.

*Transitive* – Yes as if animal a is the same species as animal b and animal b is the same species and animal c, therefore animal a must be the same species of animal c.

*Equivalence relation* – Yes as all three relations hold as the set is reflexive, symmetric and transitive.

ii)

*Reflexive* – No as negative integers would not allow this to hold. For example, for values (-2, 1), ) = (4, 1). Hence,

*Symmetric* – False

*Transitive* - True

*Equivalence* *relation* – False

iii)

*Reflexive* - False

*Symmetric* - True

*Transitive* - False

*Equivalence* *relation* - False

iv)

*Reflexive* - False

*Symmetric* - True

*Transitive* – False

*Equivalence* *relation* - False

v)

*Reflexive* - False

*Symmetric* - False

*Transitive* - False

*Equivalence* *relation* – False

b)

i)

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ii)

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Description automatically generated

iii)

A math equations on a piece of paper

Description automatically generated

iv)

As there is a bijection is the function

We can say that the cardinality of is equal to the cardinality of

Written as |

Given the theorem |

We can say that both are countably infinite.

c)

i)

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ii)

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iii)

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